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EDITED BY

W. J. GREENSTREET, M.A.

WITH THE CO-OPERATION OF

F. S. MACAULAY, M.A., D.Sc.

AND

PROF. E. T. WHITTAKER, M.A., F.R.S.

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IMAGINARIES IN GEOMETRY, AND THEIR INTERPRETATION IN TERMS OF REAL ELEMENTS.

By C. V. H. Rao, M.A.

The principal ideas are due to Von Staudt * and Lüroth.† There is also a paper by Prof. Mathews,‡ though it does not go far enough; and a recent treatise § in English on the subject makes no reference at all to the question. It may be useful therefore to put down a brief account of the leading ideas of the two writers first mentioned, together with some simple developments.

The object is to define an imaginary point as the result of a specific process applied to a finite number of real points. We may then extend the definition to higher elements. In any operation on real loci we find that if imaginaries occur at all, they occur in pairs; we may then think of two conjugate imaginary points as the common points of a line and a conic. And this is how Von Staudt began. We endeavour to get rid first of the conic and then of the line. The pairs of conjugate points on the line l set up an involution which has X, X' for double points; and the involution is defined by the double points. If A, A' is any pair, then $(AA', XX') = -1$. We notice that when X, X' are imaginary, any two pairs A, A' and B, B' separate each other, so that the sense ABA' is opposed to that of $AB'A'$. We may thus indicate X, X' by $[ABA'B']$ and $[AB'A'B]$. This representation is ∞^2, but we may modify it so as to reduce the freedom. For as soon as one pair A, A' is given we can always find a second pair B, B' such that $(AA', BB') = -1$; X, X' and A, A' and B, B' are then three mutually harmonic pairs of points. We thus arrive at a representation by four harmonic points, the representation being singly infinite.

Now the same result holds in the case of a conic, for corresponding to a projective relation between a number of points on a line there exists precisely the same relation between the points on a conic which have the same parameters on the conic as the points on the line. In particular, if S is a real conic through X, X', let O be the pole of XX'. Let AA', BB' be any two chords through O. Then both the pairs of points A, A' and B, B' are harmonic with respect to X, X'; and the line XX' is in fact the third diagonal of the

* *Geometrie der Lage*, Nuremberg, 1856, page 76.

† *Math. Annalen*, vol. 8, 1875, page 145.

‡ *Proc. London Math. Soc.* vol. 11, 1912.

§ Hatton, *The Imaginary in Geometry*, Cambridge, 1920.

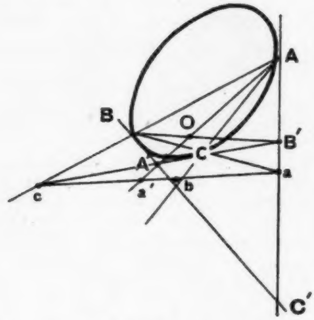
quadrilateral $ABA'B'$. Also given AA', we can find uniquely a chord BB' through O such that (AA', BB') is harmonic.

The above method is open to this objection, that to limit it so as to apply to imaginary points only we have to fall back on the proviso that the involution must be of such a nature that any two pairs separate each other. This idea of separation is a somewhat delicate one, and it would be preferable to be able to do without it. Lüroth undertook the task, and proceeds as follows.

2. Given three real points a, b, c on a line, the double points of the cyclic projectivity $abc \wedge bca$ are always imaginary, as can be seen easily from the form of the equation

$$(b-c)^2(\theta-a)^2+(c-a)^2(\theta-b)^2+(a-b)^2(\theta-c)^2=0,$$

whose roots are imaginary when a, b, c are real. Taking a conic and an inscribed triangle ABC, the tangents at the vertices meet the opposite sides in points a, b, c lying on a line l, which is the axis of the projectivity $ABC \wedge BCA$; the intersections X, X' of l with the conic are precisely the double points of the projectivity, and are given by the Hessian of the cubic form representing A, B, C.



If the tangents at A, B, C form the triangle $A'B'C'$, then the lines AA', BB', CC' concur in a point O, which is the pole of the line l. The line l and the point O may be called the axis and the centre of the triangle ABC in relation to the particular circumconic under consideration.* Projecting the range ACB on to the line l from the points A, C, B respectively, we have

$$abc \wedge bca \wedge cab;$$

and the double points are the points in which l meets the conic. Projecting ACB from any other point P of the conic on the line l, we have a set a_1, b_1, c_1, which is equivalent to the set a, b, c in the sense that $a_1b_1c_1 \wedge b_1c_1a_1$ has the same double points. The two points X, X' may then be represented by the symbols $[ABC]$, $[ACB]$ or $[abc]$, $[acb]$.

It is easy to work out the passage from the three-point representation to the four-point representation. For let AA' meet l in a'. Then, from a consideration of the quadrilateral $ABCA'$, it follows that (aa', bc) is harmonic. Also a, a' are conjugate with respect to the conic, for a being the point common to the lines BC, l, whose poles are A', O, the polar of a is OA'. Thus a, a' are conjugate and (aa', XX') is harmonic. Therefore, defining a', b', c' by the relations

$$(aa', bc)=(bb', ca)=(cc', ab)=-1,$$

* When the conic is taken as a circle, the centre of the triangle as here defined becomes what is usually known as the Symmedian point.

we find that the pair X, X' is harmonic with respect to each of the pairs of points aa', bb', cc'. Thus the points X, X', which are $[abc], [acb]$, are the same as $[aba'b']$, $[ab'a'b]$.

But it is a more difficult matter to effect the converse passage. This is effectively equivalent to the following question. Given a conic and a point O to inscribe a triangle ABC such that its centre shall coincide with O. $A'B'C'$ being the triangle formed by the tangents at A, B, C, let AA' meet the conic again in A''. Then by projecting the given conic into a circle having the projection of O for centre, we find that $(A'A''OA) = -\frac{1}{3}$. Thus A' is situated on a definite conic having double contact with the given one, where it is met by the polar of O. The determination of A' is not possible by linear constructions.

The representation by four harmonic points as well as the representation by three points are both ∞^1, and we now proceed to show that there is one representation which is in a sense a connecting link between the two. In the first place we observe that

$$(ABCX) = (BCAX) = (CABX) = -\omega,$$
$$(ABCX') = (BCAX') = (CABX') = -\omega^2,$$

where w is a cube root of unity; this can most easily be seen by projecting X, X' into the circular points at infinity. X, X' are thus the equianharmonics of any one of the points A, B, C with respect to the other two. It is also to be noticed that the fourth harmonic and the two equianharmonics of the point at infinity in regard to X, X' form a three-point representation.

Thus we have the result that the two points $\alpha \pm i\beta$ *are the double points of the cyclic projectivity*

$$\alpha - \sqrt{3}\beta,\ \alpha,\ \alpha + \sqrt{3}\beta \wedge \alpha,\ \alpha + \sqrt{3}\beta,\ \alpha - \sqrt{3}\beta,$$

and this property serves as a definition of the points $\alpha \pm i\beta$.

3. The following analysis may render the position clearer. Given two sets of three points ABC, $A'B'C'$ on a conic, there is a projectivity defined by the pairs AA', BB', CC'. If the axis of the projectivity meet the conic in X, X', then the three points like $BC' = B'C$ lie on the line XX'. This can easily be proved, and is only a generalisation of Pappus' Theorem for two lines ABC, $A'B'C'$. Further, if P, P' is the general pair of the projectivity, there exists a definite conic having double contact with the given one at X, X', and touched by all the lines like PP'. Taking X, X' for the vertices on the y-axis, the given conic may be taken as $y^2 = zx$; and the second is then

$$y^2 = hzx, \quad h = \frac{4k}{(1+k)^2},$$

and then if P is θ, we find that P' is $k\theta$.

In particular, if A', B', C' are supposed to coincide with B, C, A respectively, we get a cyclic projectivity, and the line XX' becomes the axis of the triangle ABC. If now we take the first conic as $\Sigma yz = 0$, A, B, C being the vertices of reference, the axis and the second conic are given by

$$\Sigma x = 0, \quad \Sigma(x^2 - 2yz) = 0.$$

Further, given three fixed points A, B, C, there are in general six positions for a varying point P, such that in some order or other the cross ratio of the four points equals an assigned constant ρ. But when ρ equals $0, \pm 1, \pm\infty$, and only then the six positions reduce to three counted twice. A second exceptional case* occurs when ρ equals $-\omega$ or $-\omega^2$; the six positions now reduce to two only, each counted thrice. Suppose we now define three new points ξ, η, ζ, by the relations

$$(ABC\xi) = (BCA\eta)(= CAB\zeta) = h,$$

* These two exceptional cases correspond to the vanishing of two invariants.

where h is an arbitrary constant *, and therefore ξ, η, ζ are distinct; it then follows that

$$ABC\xi\eta\zeta \wedge BCA\eta\zeta\xi,$$

so that the set ξ, η, ζ is equivalent to the set A, B, C, in the sense of having the same Hessian points, and either of the two sets is obtainable from the other by a projective shift.

4. Before passing to elements in space, we put down a number of statements, some of which must be accepted as axiomatic. Through an imaginary point there cannot pass more than one real line, and there does pass one real line, which may be called its base line. All real planes through an imaginary point have in common one real line, viz., the base line of the point. We next remark that with respect to a real conic an imaginary point in its plane reciprocates into an imaginary line. Hence on an imaginary line lying in a real plane † there exists one and only one real point, which may be called the base point, and analogously such a line may be represented by the symbol $[abc]$, where a, b, c are three real lines through the base point. Consider now the following question of construction. Given in a real plane two imaginary lines $[abc]$, $[a'b'c']$, the base points being O, O', to find the base line through the imaginary point common to the two lines. Let the lines a, b, c meet the lines a', b', c' respectively in the points A, B, C. Then it follows from § 2 that the required base line is the axis of the triangle ABC with respect to the circumconic through O, O'.

We saw above that on an imaginary line lying in a real plane there exists one and only one real point. Reciprocating with respect to a real quadric we obtain an imaginary line which contains one real point only, and through which there passes one real plane only. Such a line is called an imaginary line of the first type, and may be denoted by l_1. Further, an l_1 may be represented by a triad of real coplanar lines through its base point, the plane of the three lines being the real plane through the l_1. The base lines of the various imaginary points on l_1 all lie in this plane, and every real line in this plane is the base line of the point where it meets the l_1. We can also show that an l_1 cannot lie on a real single-sheeted hyperboloid. For in the first place the quadric must also pass through the conjugate line l_1'; and O being the common base point, the tangent plane at O, which is a real point, will be a real plane meeting the quadric in two imaginary generators, which is absurd.

Again, any imaginary plane ‡ contains one and only one real line which is its base line. The line common to two imaginary planes whose base lines meet is precisely an l_1. On the other hand, if the base lines do not meet, we obtain an imaginary line with no real point on it, and therefore also with no real plane through it. It is an imaginary line of the second type, and may be denoted by l_2. Let O, O' be the two base lines of the two imaginary planes $[abc,]$ $[a'b'c']$, where a, b, c are real planes through O, and a', b', c' are real planes through O'; and O, O' do not meet. Let the lines $a=a'$, $b=b'$, $c=c'$ be indicated by α, β, γ. α, β, γ are then skew lines, all of which are met by both O and O'. The line l_2 obviously meets both O and O'. We shall now show that the lines α, β, γ, l_2 are all generators of a quadric. For the lines α, β, γ may be represented by three parameters λ of the quadric determined by them, say λ_1, λ_2, λ_3; the line $[\alpha\beta\gamma]$ is then λ_4, where symbolically λ_4 is $[\lambda_1\lambda_2\lambda_3]$. There exists therefore at least one real ruled quadric through an l_2. And in fact we can make the following more general statement. Through

* Not equal to $-\omega$ or $-\omega^2$.

† This is a very necessary qualification, as will be apparent shortly.

‡ We are considering only imaginary planes situated in a real space of three dimensions. There are obvious generalisations. For instance, there exist imaginary planes which contain only a single real point, but such planes do not exist in a real space of three dimensions, but only in higher space.

a line l_1 there passes only one real plane, an ∞^3 of real cones, and no real ruled quadric; and through a line l_2 there pass an ∞^3 of real ruled quadrics, and no real cones nor real planes.

5. We have now a number of simple results, which may be called lemmas of existence, of which the following is a simple example. Given two triads of points ABC, $A'B'C'$ on two lines l, l' respectively, we can always find a triad $A_1B_1C_1$ on l' such that

$$A'B'C'A_1B_1C_1 \wedge B'C'A'B_1C_1A_1,$$

and such that the lines AA_1, BB_1, CC_1 are concurrent; and the new triad is unique. This is an expression of the fact that on a line l_1 there exists a real point. So also there is a slightly more complicated result expressing the fact that any imaginary plane contains a real line. But it is to be noticed that the results thus obtained are much more general than the facts in the geometry of the imaginary which they express; for in the enunciation the points are perfectly arbitrary and not limited to be real or imaginary.

There are again six fundamental constructions, two in a real plane and four in space, to which any question of construction can be reduced. One of them has already been considered in § 4.

From the above work a number of interesting results may be deduced. We give two examples.

Given three skew lines α, β, γ determining a quadric S, any plane ϖ meets them in a conic s and an inscribed triangle ABC. If l is the axis of the triangle ABC with respect to s, so also is it the axis of $A'B'C'$ with respect to s', where A', B', C', s' are the traces of the given lines and quadric by any other plane ϖ' through l.

By way of proof, we content ourselves with pointing out that $[\alpha\beta\gamma]$ is an imaginary generator of the quadric and l is the base line of the point where it meets ϖ. The second example is as follows.

Given three chords AA', BB', CC' of a conic s, there exists in general a single conic σ, which, besides having double contact with s, touches the three chords in, say, the points α, β, γ. If now l, l' denote the axes of the triangles ABC, $A'B'C'$ with respect to s, then one of the diagonal points of the quadrilateral fixed by $s=l$, $s=l'$ is the centre of the triangle $\alpha\beta\gamma$ with respect to σ, and the other lies on the chord of contact.

The latter part of the theorem is easy to prove geometrically, and the first part may be left as an exercise. C. V. H. RAO.

GLEANINGS FAR AND NEAR.

56. Barnes, the Dorsetshire poet, read mathematics with General Shrapnel, the inventor of the shell. One of the poet's pupils, whom he prepared in mathematics and Hindostanee, came out first in the Indian Civil Service list. In 1837 Barnes entered his name at St. John's as a "ten year man." At the end of his probationary period, and after a residence of two or three terms and the performance of certain exercises, the ten year man proceeded to his B.D. degree. Ten year men did not graduate in Arts.

57. The *stock history* of the rise of geometry—the Egyptians inventing an art of land-surveying in order to preserve the memory of the bounds of property—with another story attributing the science directly to the gods, "forms the first light we have on the subject, and both in one are worthily sung by the poet who figures at the head of an obsolete English course of mathematics:

> 'To teach weak mortals property to scan
> Down came geometry, and formed a plan.'"

[What course ?]

MATHEMATICAL NOTES.

548. [R. L.[1]] Note 538. *The Sound Ranging Problem.*

This is an application of the geometrical problem of determining a circle to touch three given circles; a full description of the method will be found in the *Constructive Geometry of Plane Curves*, by T. H. Eagles, 1885, pp. 46, 49.

The method given there by Eagles would be rather long to employ in the field, so the following modification is suggested, capable of all the accuracy required for the location of a gun P, where the time, t_1, t_2, t_3 seconds, when the report of the gun is heard at the station, S_1, S_2, S_3, is noted, from an epoch just before the gun is fired, say at the flash if visible.

Draw the three circles, centre at S_1, S_2, S_3, and radius the distance the sound travels in the time t_1, t_2, t_3; reduced to scale on the map.

The unit of length here may be called the *sound-second*, by analogy with the *light-year* of Astronomy. Over the ground the normal sound-second would be about 1140 feet, 380 yards, 350 metres, requiring correction for the temperature in accurate measurement.

In Astronomy, the light-year would be

$$365\tfrac{1}{4} \times 24 \times 60 \times 60 \times 300{,}000 \text{ kilometres, say } 10^{12} \times 9{\cdot}464 \text{ km.}$$

The gun is then at P, the centre of a circle touching these three circles.

There are eight circles in all to choose from, one touching the three circles all externally, one touching all internally, three touching one circle externally and the other two internally, and the last three touching one circle internally and the other two externally.

In the Sound Ranging problem, the choice will lie between the first two circles, as we suppose t_1, t_2, t_3 positive.

Any alteration of the epoch will increase or diminish the three radii by an equal amount, and so will leave the centre P unaltered.

If the epoch is chosen at the point where the sound of P is first heard, say at S_1, $t_1=0$, and the geometrical problem is to draw a circle passing through S_1, and touching the two circles round S_2 and S_3; this is the simplified case considered by Eagles on p. 46.

But if S_2 is where the sound is heard next, and the epoch is taken there, making $t_2=0$, then t_1 is negative, and the complication is introduced of a circle touching the S_1 circle internally and S_3 circle externally, and passing through S_2; confusion would then be likely to arise in practice.

The method is required principally for the exact location of a gun in concealment. But if the flash was visible and taken as the epoch, the three circles round S_1, S_2, S_3 should intersect at P over the gun, on the assumption of perfect measurement.

The inevitable errors of observation will cause the three circles to form a triangle instead, enclosing P, at the centre of the inscribed circle, and then P can be fixed with accuracy by divers obvious methods.

In an actual case the position of the gun would be inferred from the direction of the sound, near enough to locate it approximately; and then to fix it with complete accuracy, the three circles round S_1, S_2, S_3 must be increased in radius by equal amount till they intersect, or very nearly, making a small triangle enclosing P; after which it is easy to increase the radii still further till they are seen by eye to intersect in P on the map. The method should prove rapid, and of greater accuracy than any approximation where the hyperbola is replaced by an asymptote, as described in the note by G. H. Bryan.

Staple Inn, *May* 1, 1920. G. GREENHILL.

549. [D. 2. b.] *Summation of Harmonic Progressions.*

In the standard textbooks on mathematics it is stated that no general method has been discovered for giving the sum of a harmonic progression, other than that of direct addition of all the terms.

In Carr's *Synopsis of Pure Mathematics* (1886), an approximate formula for the calculation of such a sum is given.

According to this formula, the sum of n terms of the H.P. $(a+d)^{-1}$, $(a+2d)^{-1}$, $(a+3d)^{-1}$, etc., when d is small compared with a, is

$$\{(a+d)^n - a^n\}/\{d(a+d)^n\}.$$

In practice this formula gives large errors.

The following is a formula which gives very close approximations to the sums of progressions of this kind to any given number of terms.

Considering a hyperbola having the equation $xy=1$ when referred to rectangular co-ordinates, it is obvious that if abscissae equal to a, b, c, d, etc., are taken in arithmetical progression, the corresponding ordinates on the curve are a^{-1}, b^{-1}, c^{-1}, d^{-1}, etc., and are in harmonic progression.

If now ordinates are raised to the curve corresponding to abscissae equal to $a-\frac{1}{2}d$ and $z+\frac{1}{2}d$ (where a and z represent respectively the smallest and largest terms of the related arithmetical progression and δ its common difference), the area enclosed between the curve, these ordinates and the axis of reference may be found by integration.

This gives
$$\text{Area} = \int_{(a-\frac{1}{2}\delta)}^{(z+\frac{1}{2}\delta)} \frac{dx}{x}$$
$$= \{\log_{10}(z+\tfrac{1}{2}\delta) - \log_{10}(a-\tfrac{1}{2}\delta)\}/(0\cdot4343).$$

If the value of the area so found be divided by "δ," the height of a long rectangle equivalent to the area under the curve is obtained.

The height of this rectangle closely approximates in magnitude to the sum of the harmonic progression.

The complete formula so deduced is:

$$S = \{\log_{10}(z+\tfrac{1}{2}\delta) - \log_{10}(a-\tfrac{1}{2}\delta)\}/(0\cdot4343d),$$

where s is the sum of the harmonical progression and a, z, and δ are as stated.

As an example, if 21 terms of the progression $10^{-1}+11^{-1}+12^{-1}+\dots$ are taken, the sum found by addition $=1\cdot1660179$.

The sum found by Carr's formula is $0\cdot891587$, giving a $-$error$=23\cdot6$ % of the true value. The sum found by the above formula $=1\cdot16643$ gives a $+$error of $\cdot036$ % of the true value.

Although in all cases a much closer approximation is reached by using the latter formula, the error in this is also variable.

For all series having the same ratio d/a and the same number of terms, the percentage error is the same.

The error is larger with series having a larger d/a ratio and diminishes with the number of terms, but not in direct proportion.

It would be interesting to learn if this or similar formulae have been previously given. W. J. Rickets.

550. [K[1]. δ. a.] *Some Propositions relative to a Tetrastigm.*

Definitions. If from a point D perpendiculars DX, DY, DZ be drawn respectively to the sides BC, CA, AB of a triangle ABC, then

(1) XYZ is called the Pedal Triangle of D with respect to ABC,

(2) the circle XYZ is called the pedal circle of D with respect to ABC.

Notation.

With a figure of four points A, B, C, D there are four pedal triangles, one for each point with respect to the triangle formed by the remaining three.

The following notation is adopted :

The feet of perpendiculars drawn to BC are marked X,

" " AD " X',

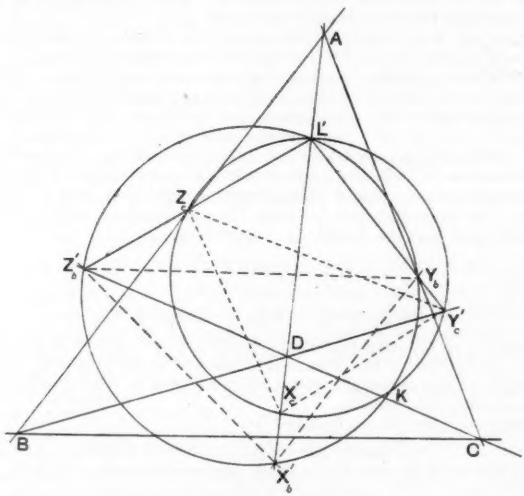
and cyclically for the other connectors.

Thus the pedal triangle of A with regard to BCD is $X_a Y_a' Z_a'$,

" " B " CDA " $X_b' Y_b Z_b'$,

etc., the suffix indicating the point from which the perpendicular is drawn and the letter indicating the line to which it is drawn.

E.g. Z_d denotes the foot of the perpendicular drawn from D to CA.



Prop. 1. The four pedal triangles are similar.

(It can easily be proved in each case that

$$YZ : ZX : XY = BC\,.\,AD : CA\,.\,BD : AB\,.\,CD.)$$

If the point D be taken inside the triangle ABC, the following angular relations hold good :

(1) $\hat{X} = B\hat{D}C - B\hat{A}C$; (2) $\hat{Y} = C\hat{D}A - C\hat{B}A$; (3) $\hat{Z} = A\hat{D}B - A\hat{C}B$.

On drawing the figure of the pedal triangles of B and C, say, it will be found that the similarity is direct.

The ratio of similarity of any two admits of an easy expression : thus

$$K_{bc} = \frac{X_b Z_b'}{X_c' Z_c} = \frac{BD \sin CDA}{CA \sin DAB} = \frac{BD\,.\,CA}{2R_b} \cdot \frac{2R_c}{CA\,.\,BD} = \frac{R_c}{R_b},$$

where R_b denotes the radius of the circumcircle of the triangle formed by leaving out B from the four points.

If now P_a denote the radius of the pedal circle of A, etc., since

$$P_b / P_c = K_{bc} = R_c / R_b,$$

it follows that $$P_a R_a = P_b R_b = P_c R_c = P_d R_d.$$

Prop. 2. The pedal circles of any two of the points (BC) intersect

(1) at a point (L') on the join of the remaining two (AD), this point being a centre of perspective of the pedal triangles of the first two (BC);

(2) at a point (K) which is the centre of similitude of these pedal triangles.

(1) Let $Z_b'Z_c$ intersect AD in L'.

$BZ_b'Z_cY_bY_c'C$ is cyclic ; (BC diameter)

$\therefore\ \angle Z_cZ_b'Y_b = \angle Z_cBY_b$, *i.e.* $\angle ABY_b$.

$BX_b'Y_bA$ is a cyclic quad. ; (BA diameter)

$\therefore\ \angle ABY_b = \angle AX_b'Y_b$.

$\therefore\ \angle Z_cZ_b'Y_b = \angle AX_b'Y_b$,

i.e. $\angle L'Z_b'Y_b = \angle LX_b'Y_b$;

$\therefore$ circle $X_b'Y_bZ_b'$ passes through L'.

$\therefore\ \angle X_b'Y_bZ_b' = \angle X_b'L'Z_b'$.

But $\angle X_b'Y_bZ_b' = \angle X_c'Y_c'Z_c$; (Prop. 1)

$\therefore\ \angle X_c'Y_c'Z_c = \angle X_b'L'Z_b'$,

i.e. $\angle X_c'Y_c'Z_c = \angle X_c'L'Z_c$;

$\therefore$ circle $X_c'Y_c'Z_c$ passes through L'.

Hence $\angle X_b'L'Y_b = \angle X_b'Z_b'Y_b$

and $\angle X_c'L'Y_c' = \angle X_c'Z_cY_c'$.

But $\angle X_b'Z_b'Y_b = \angle X_c'Z_cY_c'$; (Prop. 1)

$\therefore\ \angle X_b'LY_b' = \angle X_c'L'Y_c'$;

$\therefore\ L'$, Y_b and Y_c' are collinear,

i.e. the pedal triangles $X_b'Y_bZ_b'$, $X_c'Y_c'Z_c$ are in perspective at L'.

(2) It is easy to prove $\angle KX_b'X_c' = \angle KY_bY_c' = \angle KZ_b'Z_c$

and $\angle KX_c'X_b' = \angle KY_c'Y_b = \angle KZ_cZ_b'$.

These equations shew that K is the centre of similitude of the pedal triangles of B and C.

Prop. 3. The four pedal circles pass through the point which is common to the four nine-point circles of the four triangles, which can be formed by taking every three of the four points.

Perhaps the easiest manner of proof is as follows:

Consider the point D and the triangle ABC.

Let the reflexions of D with regard to BC, CA, AB be respectively D_a, D_b, D_c.

By angular relations it is easy to prove that the circles BD_aC, CD_bA, AD_cB are concurrent at a point E which is on the circle $D_aD_bD_c$.

Now take D as an origin from which all vectors drawn are bisected, and we find at once that

The nine-point circles of BDC, CDA, ADB meet at a point K (the middle point of DE) which is on the pedal circle of D with regard to ABC.

Since the point of concurrence of three nine-point circles must be the unique point of concurrence of the four, the above statement follows.

From this and Prop. 2(2) we conclude that the four pedal triangles have a common centre of similitude at the point K.

We might call this point K the "nine-circle point" of the quadrilateral $ABCD$, because it is a well-known theorem that the circle through the intersections (BC, AD), (CA, BD), (AB, CD) passes through the point common to the four nine-point circles. Calling this latter circle the harmonic circle of the quadrilateral, we have K as the point of concurrence of

(1) The four nine-point circles.
(2) The four pedal circles.
(3) The harmonic circle.

The name "Nine-Circle Point" when compared with "Nine-Point Circle" suggests a system of reciprocation by which points might be trans-

formed into circles; but, whereas the nine points are grouped into three triads and the nine circles are not, it does not seem likely that such a system would derive one property from the other.

K is of course the centre of the equilateral hyperbola which passes through $ABCD$ and is indeterminate when these points form an orthocentric system.

We now see that the pedal circle of D with regard to ABC intersects the nine-point circle of ABC at a point which is uniquely related to the figure $ABCD$.

The question is suggested, what is the other point of intersection? This may be answered as follows:

Let D' be the isogonal conjugate of D with regard to ABC.

The pedal circle of D' is the same as that of D (in fact, every pedal circle has what we may call its two poles).

$\therefore$ the pedal circle of D also meets the nine-point circle of ABC at a point which is uniquely related to the figure $ABCD'$.

If D and D' coincide, these points of intersection coincide, and the pedal circle touches the nine-point circle (Feuerbach's Theorem).

The following propositions, some of which are well-known, are based upon the foregoing.

1. $ABCD$ are four concyclic points.

The Simson line of each point with respect to the triangle formed by the remaining three is drawn.

Prove that (α) the Simson lines are divided similarly by the sides of the corresponding triangle.

(β) The Simson lines are concurrent at the point common to the four nine-point circles of the four triangles.

2. ABC is a triangle, LMN the feet of any three concurrent bisectors of the angles, I the point of concurrence.

(1) The pedal circle of I (an inscribed or exscribed circle) touches the nine-point circle of ABC.

(2) The circle LMN passes through the point of contact.

(3) The nine-point circles of BIC, CIA, AIB pass through the point of contact.

3. A' is the isogonal conjugate of A with regard to BCD, etc.

(1) $B'C'$ is perpendicular to AD.

(2) $B'C'$ meets AD at the point L', in which the pedal circles of B and C also intersect.

4. The intersections $\{X_b'Y_b, X_c'Z_c\}$ and $\{X_bZ_b', X_c'Y_c'\}$ are collinear with the middle point of BC, and the line of collinearity is perpendicular to AD.

5. P and Q are points inverse with regard to the circumcircle of ABC.

(1) The pedal triangles of P and Q are similar.

(2) The sides of ABC subtend similar angles at P' and Q', the isogonal conjugates of P and Q.

6. The nine-point circle of a triangle bisects the distance between the isogonal conjugates of points inverse with regard to the circumcircle.

(α) Derive the well-known property that the nine-point circle bisects the line joining the orthocentre to any point on the circumcircle.

7. The sides of a variable triangle touch a fixed ellipse, and an equilateral hyperbola passes through the vertices of the triangle and a focus of the ellipse.

The locus of the centre of the hyperbola is the major auxiliary circle of the ellipse.

8. The common chord of the pedal circles of B and C passes through the intersection of Y_bZ_b', $Y_c'Z_c$ and bisects each of the angles $Y_c'KZ_b'$, Y_bKZ_c.

9. The sides of ABC subtend similar angles at D and E.

The nine-circle points of $ABCD$ and $ABCE$ coincide.

10. It is probably true that

The common chord KL' of the pedal circles of B and C bisects the upper segments of the perpendiculars (1) from A in the triangle ABC, (2) from D in the triangle DBC.

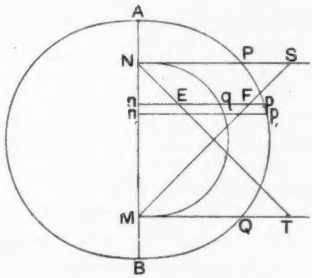
If this is established it will help to prove that

The lines joining A to the orthocentre BCD and D to the orthocentre ABC intersect on the common chord of the pedal circles of B and C.

The College, Maritzburg. JOHN H. LAWLOR.

551. [K[1]. 16. g.] *The Volume of a Frustum of a Sphere.*

Let the frustum be generated by the revolution of the arc PQ about the diameter AB.



Through P and Q draw ordinates NP and MQ, and cut off NS and MT each equal to MN.

Let any ordinate pn meet NT in E, MS in F and the semicircle on NM in q. Let n_1p_1 be an ordinate at a small distance δx from np.

$$pn^2 = An\,.\,nB = (AN+Nn)(nM+MB)$$
$$= AN\,.\,nM + AN\,.\,MB + Nn\,.\,MB + Nn\,.\,nM.$$

But $\quad Nn\,.\,nM = qn^2;$

$$\therefore\ pn^2 - qn^2 = AN\,.\,MB + AN\,.\,nM + Nn\,.\,MB$$

and $\pi pn^2.\delta x - \pi qn^2.\delta x = \pi\{AN\,.\,MB\,.\,\delta x + AN\,.\,\overline{nM\,.\,\delta x} + MB\,.\,\overline{Nn\,.\,\delta x}\}$

$$= \pi\{AN\,.\,MB\,.\,\delta x + AN\,.\,\overline{nF\,.\,\delta x} + MB\,.\,\overline{nE\,.\,\delta x}\}.$$

Summing the thin slices, which make up the volume of frustum − volume of the sphere on NM as diameter, we get

$$\pi AN\,.\,BM\,.\,NM + \pi AN\,.\,\triangle NSM + \pi MB\,.\,\triangle NMT$$
$$= \pi AN\,.\,BM\,.\,NM + \tfrac{1}{2}\pi AN\,.\,NM^2 + \tfrac{1}{2}\pi MB\,.\,NM^2$$
$$= \tfrac{1}{2}\pi NM\{2AN\,.\,BM + AN\,.\,MN + MB\,.\,MN\}$$
$$= \tfrac{1}{2}\pi NM\{AN(BM+MN) + BM(AN+MN)\}$$
$$= \tfrac{1}{2}\pi NM\{AN\,.\,NB + BM\,.\,AM\}$$
$$= \tfrac{1}{2}\pi MN(PN^2 + QM^2)$$

$= \frac{1}{2}$ sum of cylinders of height MN and radii PN and QM;

$\therefore$ the volume of the frustum

= the volume of sphere on the height of the frustum as diameter + $\frac{1}{2}$ sum of cylinders on the ends of the frustum as bases, and having the same height.

N.B.—The procedure fails to give the volume of a frustum, which is the whole sphere. F. W. RUSSELL.

552. [K^1. 6. b.] *On the Conic in Polar Co-ordinates.*

We propose to obtain, from geometrical considerations, certain results in polar co-ordinates, with the focus of the conic as pole.

1. *The tangent at a given point.*

Taking any point R (Fig. 1) whose co-ordinates are r, θ, on the tangent

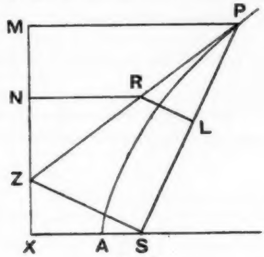


FIG. 1.

at P, whose vectorial angle is α, and drawing RL, RN perpendicular to SP and the directrix respectively, we have, since PSZ is a right angle,

$$\frac{SL}{SP}=\frac{ZR}{ZP}=\frac{RN}{PM};$$

$$\therefore\ SL/RN=SP/PM=e.$$

Hence $$r\cos(\theta-\alpha)=e(SX-r\cos\theta)=l-er\cos\theta;$$

$$\therefore\ l/r=e\cos\theta+\cos(\theta-\alpha).$$

2. *The chord through two given points.*

With the usual notation, let $\alpha-\beta$ and $\alpha+\beta$ be the vectorial angles of the points P and Q (Fig. 2), the chord through which meets the directrix in Z.

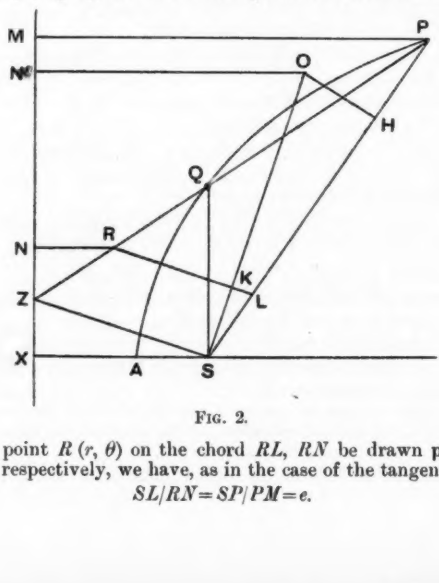


FIG. 2.

If from any point R (r, θ) on the chord RL, RN be drawn parallel to SZ and the axis respectively, we have, as in the case of the tangent,

$$SL/RN=SP/PM=e.$$

But RL is perpendicular to SK, the internal bisector of $\angle PSQ$ ($\angle PSQ = 2\beta$), and $\angle KSX = \alpha$. Hence

$$SR\cos(\theta-\alpha) = SK = SL\cos\beta\ ;$$

$$\therefore\ r\sec\beta\cos(\theta-\alpha) = e\,.\,RN = l - er\cos\theta,$$

that is,

$$l/r = e\cos\theta + \sec\beta\cos(\theta-\alpha).$$

3. *The polar, or chord of contact of tangents from a given point.*

Let PQ (Fig. 2) be the chord of contact of tangents from a given point O (r', θ'), and from O draw OH, ON', perpendicular to SP and the directrix respectively.

Then, since

$$SH/ON' = SL/RN = e,$$

we have

$$e^2\,.\,RN\,.\,ON' = SL\,.\,SH = SK\,.\,SO.$$

Hence, at once, $(l - er\cos\theta)(l - er'\cos\theta') = rr'\cos(\theta-\theta')$.

4. *The normal at a given point.*

Taking any point Q (Fig. 3), whose co-ordinates are r, θ, on the normal

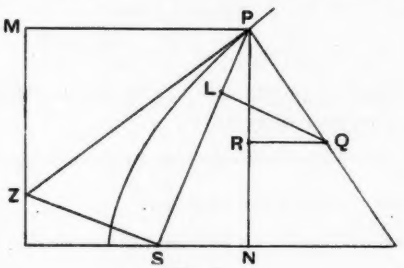


FIG. 3.

at P, whose vectorial angle is α, and drawing QL, QR perpendicular to SP and the ordinate PN respectively, we have, by similar triangles,

$$\frac{QL}{SP} = \frac{PQ}{PZ} = \frac{PR}{PM};$$

$$\therefore\ QL/PR = SP/PM = e.$$

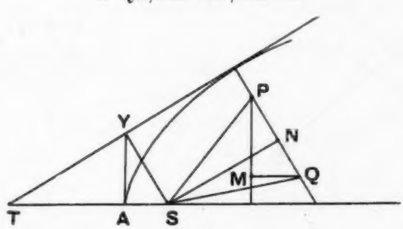


FIG. 4.

Hence

$$r\sin(\theta-\alpha) = e\,.\,PN - er\sin\theta\ ;$$

$$\therefore\ er\sin\theta + r\sin(\theta-\alpha) = e\,.\,SP\sin\alpha$$
$$= el\sin\alpha/(1 + e\cos\alpha),$$

the required equation to the normal. ANON.

553. [R. 7. a.] *A Method of finding the Normal Acceleration in Circular Motion.*

(1) Suppose a particle to describe the perimeter of the square $ABCD$ with uniform speed v.

Then at each corner of the square it must experience a change of velocity

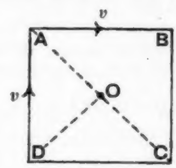


FIG. 1.

represented by $v\sqrt{2}$ and directed towards the centre of the square.

Now let O be the centre and let $OA=r$.

Join OD, then $\triangle$s DOA and ACD are similar.

$$\therefore \frac{CA}{AD}=\frac{AD}{AO} \quad \text{or} \quad CA=\frac{AD^2}{AO}=\frac{AD^2}{r}.$$

Hence, if we elect to represent velocity v by DA, the change of velocity at corner A may be represented by AC, *i.e.* by $\frac{v^2}{r}$.

(2) Take the case of a particle describing a regular polygon of n sides with uniform speed v.

Let O be centre of polygon and let $OA=r$.

Draw $HX \parallel$ to AB.

Then $HX=AB=HA$ and $\triangle$s AXH and AHO are similar.

Hence
$$AX=\frac{AH^2}{AO}.$$

The change of velocity at the corner A can be represented by AX if we represent initial and final velocities by HA and AB respectively.

Thus the change in turning the corner $A=\frac{v^2}{r}$, just as in case of square.

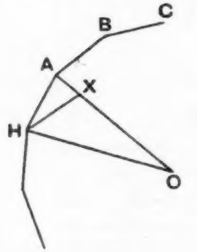


FIG. 2.

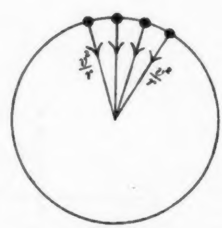


FIG. 3.

(3) Increase the number of corners indefinitely. We then get the particle describing a circle with uniform speed v, and experiencing at each point of its path a change of velocity $=\frac{v^2}{r}$ directed towards the centre of the circle.

In this case the acceleration can be measured by the change of velocity at any instant, and it therefore is $\frac{v^2}{r}$ directed towards the centre of the circle.

CYMRIC.

554. [K[1]. 1. d.] *Area of a Triangle in Terms of the Coordinates of its Angular Points.*

In the ordinary text-books of Analytical Geometry the determinantal form, into which the expression for the area is translated, plays no rôle in the investigation. The following method is free from this objection.

Let $A(x_1y_1)$, $B(x_2y_2)$, $C(x_3y_3)$ be the angular points of the triangle.

Assuming that $ax+by+c=0$ represents a straight line, we have for the line through A and B,

$$\begin{aligned} ax\ +by\ +c&=0,\\ ax_1+by_1+c&=0,\\ ax_2+by_2+c&=0; \end{aligned}$$

whence, eliminating a, b, c,

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix}=0,$$

the equation to AB.

The perpendicular, p, from C to AB is, by the ordinary rule,

$$\begin{aligned} p&=\begin{vmatrix} x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} \div\sqrt{(y_1-y_2)^2+(x_1-x_2)^2}\\ &=\begin{vmatrix} x_3 & y_3 & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} \div AB. \end{aligned}$$

Thus, the area $=\frac{1}{2}\,.\,p\,.\,AB$

$$=\tfrac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}.$$

The assumption that the linear equation represents a straight line is often justified by showing that the area of the triangle formed by any three points on the locus is zero. For this mode of proof the following may now be substituted.

Let $P(x_1y_1)$ be any selected point on the locus of $ax+by+c=0$, and $Q(x_1+h,\ y_1+k)$ any other point on the locus.

Then $$ax_1+by_1+c=0$$

and $$a(x_1+h)+b(y_1+k)+c=0,$$

whence $$ah+bk=0 \quad \text{or} \quad \frac{k}{h}=-\frac{a}{b}.$$

But $\frac{k}{h}=\tan\theta$, where θ is the inclination of PQ to the x-axis. Thus, this angle is the same for all positions of Q. The locus is consequently a straight line.

AUSTRAL.

555. [K[1]. 8. a.] *Remarks on Note* 508 : *Brocard Points for a Quadrilateral.*

In a brilliant series of articles (pp. 202, 217, 241, 265) in *Mathesis*, vol. v. 1885, Prof. Neuberg has given a large number of properties of the harmonic

quadrilateral in a circle, including the theorem obtained independently by Mr. F. G. W. Brown. A neater formula for ω is found, viz.,

$$\operatorname{cosec}^2 \omega = \operatorname{cosec}^2 A + \operatorname{cosec}^2 B,$$

where A, B are consecutive angles of the quadrilateral.

By Geometry, or the Complex, or Grassmann's methods, other results may be obtained.

If I be the intersection of BD, AC; M, N their mid-points, then a Brocard point X lies on the circles MAD, MBC, MNI.

If λ denote the ratio $MA : MD\,(=MD : MC)$,

$$\cot \omega = \frac{1}{2}\left(\lambda + \frac{1}{\lambda}\right) \operatorname{cosec} AMD \text{ (or } \hat{B}\text{)}.$$

$$MX \sin B = \sin(B+\omega) - \lambda \sin \omega$$
$$= \frac{1}{\lambda} \sin \omega - \sin(B-\omega).$$

If BD be a fixed diameter of the circle and A, C vary, the locus of X is a lemniscate.

If AB be fixed, $A\hat{C}B = A\hat{D}B = \gamma$; then

$$\cos A \cos B = \cos(A+B+2\gamma),$$

and, if P be the intersection of AD, BC, the locus of P is a parabola, ellipse or hyperbola, according as $\gamma = <$ or $> 45°$.

R. W. Genese.

556. [R. 2. b.] C.G. *of a Quadrilateral Lamina.*

Let $A_1A_2A_3A_4$ be a quadrilateral lamina of uniform density, and let the masses of the triangles $A_2A_3A_4$, $A_3A_4A_1$, $A_4A_1A_2$ and $A_1A_2A_3$ be $3m_1$, $3m_2$, $3m_3$ and $3m_4$ respectively, so that $m_1+m_3 \equiv m_2+m_4 = M$, where M is one-third of the mass of the quadrilateral.

Drawing the diagonal A_2A_4 and replacing the triangles so formed by equivalent masses at the vertices, those for the quadrilateral are m_3 at A_1, m_1 at A_3, m_1+m_3 at A_2 and A_4. Taking an identical quadrilateral and drawing the diagonal A_1A_3, the equivalent masses for the quadrilateral are also m_4 at A_2, m_2 at A_4, m_2+m_4 at A_1 and A_3. Superimposing these quadrilaterals, we have a new quadrilateral of mass $6M$ and with the following set of equivalent masses:

$$m_2+m_3+m_4 \text{ at } A_1,\quad m_3+m_4+m_1 \text{ at } A_2,\quad m_4+m_1+m_2 \text{ at } A_3,\quad m_1+m_2+m_3 \text{ at } A_4,$$
$$\textit{i.e. } 2M-m_1 \text{ at } A_1,\quad 2M-m_2 \text{ at } A_2,\quad 2M-m_3 \text{ at } A_3,\quad 2M-m_4 \text{ at } A_4.$$

The two sets of negative masses m_1 at A_1, m_3 at A_3 and m_2 at A_2, m_4 at A_4 are each equivalent to a negative mass M at the intersection of the diagonals, for this point divides each diagonal in the inverse ratio of the areas of the triangles on each side of the other diagonal.

Hence the centre of gravity of the quadrilateral lamina is the same as that of four masses, each one-third that of the lamina, placed at the vertices, together with an equal negative mass placed at the intersection of diagonals; and the distance of the C.G. from any line is $\frac{1}{3}(x_1+x_2+x_3+x_4-x_5)$, where the x's are the distances of the above points, in order, from the given line.

E. H. Smart, M.A.

557. [J. 2. f.] *Probability and Athletic Sports.*

If more than 6, and less than 13, runners enter for a race, it is usual to arrange them by lot in two heats: the first three in each heat to start in the final. If there are more than 12 runners, but less than 19, they run in three heats, the first two in each heat being chosen for the final, which contains in any case 6 runners, supposed to be the best 6.

It is commonly recognised that there is an element of chance involved in this system; but many, I think, will be as surprised as I was to find how very largely that element figures. R_n being the n^{th} best runner, the following table gives each runner's chance of reaching the final. The first column is the total number of competitors: the last is the probability that the six chosen for the final are actually the best six.

PROBABILITY OF REACHING THE FINAL.

Number of Runners.	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}	R_{11}	R_{12}	R_{13}	R_{14}	Best Six.
7 or 8	1	·971	·886	·714	·429	0	·	·	·	·	·	·	·571
9 or 10	1	·952	·833	·643	·405	·167	0	·	·	·	·	·	·476
11 or 12	1	·939	·803	·608	·392	·197	·061	0	0	0	·	·	·433
13	·937	·831	·698	·555	·413	·282	·171	·085	·028	0	0	·	·210
14 or 15	·934	·824	·689	·545	·406	·280	·175	·095	·041	·011	0	0	·200
16	·929	·813	·676	·533	·399	·280	·182	·107	·054	·022	·005	0	·187
17 or 18	·926	·809	·670	·528	·395	·278	·183	·111	·060	·028	·010	·002	·182

Some points worth noticing in this table are:

(1) The discontinuity at the change from two to three heats is of a markedly Bolshevist type, and enriches all the inferior runners suddenly at the expense of the best six.

(2) If any horizontal row be plotted graphically, no discontinuity is found, even between R_6, who deserves to be in the final, and R_7, who does not.

(3) R_7's chance is never less than $\frac{3}{5}$ of R_6's, and is generally about $\frac{3}{4}$ of it.

(4) The odds are almost always against the six chosen being the best six, and are even as heavy as 9 to 2 against, if there are 18 runners.

I think the conclusion to be drawn is that mathematical masters ought to regard with some suspicion any system of awarding prizes, or marks in an Athletic Competition, on 5th or 6th places in races which are run in heats.

The reader may amuse himself by drawing the graph for the probability of reaching the final, when 18 runners race in 3 heats.

Eton College. W. HOPE-JONES.

SOME INCIDENTAL WRITINGS BY DE MORGAN.

XVII.

(*Continued from* p. 74, vol. x.)

There is a great deal of good answering to tolerably rational questions, mixed with some attempts at humour, and other eccentricities, and occasionally a freedom, both of question and answer, by which we might, were it advisable, confirm the fact, that the decorums of 1736 and of 1850 are two different things.

First, as an instance of question and answer, which might do as well (if the record be correct) for the present publication.

"*Q*. We read in our public papers of the Pope's Bull and the Pope's Brief; pray, Gentlemen, what is the difference between them ?

"*A*. They differ much in the same manner as the Great Seal and Privy Seal do here in England. The Bull being of the highest authority where the papal power extends ; the Brief is of less authority. The Bull has a leaden seal upon silk affixed to the foot of the instrument, as the wax under the Great Seal is to our letters patent. The Brief has *sub annulo piscatoris* upon the side."

Query. Is this answer complete and correct ?

Now for another specimen :

"*Q*. Wise Oracle show,
A good reason why,
When from tavern we *go*,
You're wel*come* they cry.

"*A*. The reason is plain,
'Cause doubtful to know,
Till seeing their gain,
If you *came well* or no."

The following is an example of unanswerable refutation. To show why a man has not one rib less than a woman, it is stated that imperfections are not hereditary ; as in the case of

"One Mr. L——, an honest sailor not far from Stepney, who has but one arm, and who cannot walk himself without the assistance of a wooden leg, and yet has a son, born some years after the amputation of his own limbs, whom he has bred both a fiddler and a dancing master."

One more, not for the wretched play upon words, but because it may make a new Query,—What does it all mean ?

"*Q*. Gentlemen, in the preamble to the late Earl of Oxford's patent, I observed, 'And whom they have congratulated upon his escape from the rage of a flagitious parricide.' I desire to know by whom, at what time, and in what manner, the said parricide was to have been committed.

"*A*. Was to have been ! He actually was committed—to Newgate."

So much for some of the "Notes and Queeries" (as the word ought to be spelt) of a century ago. M.

"Corkscrew" (p. 242) replied : "There is nothing new under the sun," quoth the Preacher ; and such must be said of "Notes and Queries." Your contributor "M" has drawn attention to the *Weekly Oracle*, which in 1736 gave forth its responses to the inquiring public ; but, as he intimates, many similar periodicals might be instanced. Thus, we have *Memoirs for the Ingenious*, 1693, 4to., edited by I. de la Crose ; *Memoirs for the Curious*, 1701, 4to. ; *The Athenian Oracle*, 1704, 8vo. ; *The Delphick Oracle*, 1720, 8vo. ; *The British Apollo*, 1740, 12mo. ; with several others of less note. The three last quoted answer many

singular questions in theology, law, medicine, physics, natural history, popular superstitions, &c., not always very satisfactorily or very intelligently, but still, often amusingly and ingeniously. *The British Apollo: containing two thousand Answers to curious Questions in most Arts and Sciences, serious, comical, and humourous*, the fourth edition of which I have now before me, indulges in answering such questions as these: "How old was Adam when Eve was created?—Is it lawful to eat black pudding?—Whether the moon in Ireland is like the moon in England?—Where hell is situated?—Do cocks lay eggs? &c." In answer to the question, "Why gaping is catching?" the Querists of 1740 are gravely told:

"Gaping or yawning is infectious, because the steams of the blood being ejected out of the mouth, doth infect the ambient air, which being received by the nostrils into another man's mouth, doth irritate the fibres of the hypogastric muscle to open the mouth to discharge by expiration the unfortunate gust of air infected with the steams of blood, as aforesaid."

The feminine gender, we are further told, is attributed to a ship,

"Because a ship carries burdens, and therefore resembles a pregnant woman."

But as the faith of 1850 in *The British Apollo*, with its two thousand answers, may not be equal to the faith of 1740, what dependence are we to place in the origin it attributes to two very common words, a *bull*, and a *dun*?

"Why, when people speak improperly, is it termed a bull? It became a proverb from the repeated blunders of one *Obadiah Bull*, a lawyer of London, who lived in the reign of Henry VII."

Now for the second:

"Pray tell me whence you can derive the original of the word *dun*? Some falsely think it comes from the French, where *donnez* signifies *give me*, implying a demand of something due; but the true original of this expression owes its birth to one *Joe Dun*, a famous bailiff of the town of Lincoln, so extremely active, and so dexterous at the management of his rough business, that it became a proverb, when a man refused to pay his debts, 'Why don't you *Dun* him?' that is, why don't you send Dun to arrest him? Hence it grew a custom, and is now as old as since the days of Henry VII."

Were these twin worthies, Obadiah Bull the lawyer, and Joe Dun the bailiff, men of straw for the nonce, or veritable flesh and blood? They both flourished, it appears, in the reign of Henry VII.; and to me it is doubtful whether one reign could have produced two worthies capable of cutting so deep a notch in the English tongue.

"To dine with Duke Humphrey," we are told, arose from the practice of those who had shared his dainties when alive being in the habit of perambulating St. Paul's, where he was buried, at the dining time of day; what dinner they then had, they had with Duke Humphrey the defunct.

Your contributor, Mr. Cunningham, will be able to decide as to the value of the origin of Tyburn here given to us:

"As to the antiquity of Tyburn, it is no older than the year 1529; before that time, the place of execution was in *Rotten Row* in *Old Street*. As for the etymology of the word *Tyburn*, some will have it proceed from the words *tye* and *burn*, alluding to the manner of executing traitors at that place; others believe it took its name from a small river or brook once running near it, and called by the Romans Tyburnia. Whether the first or the second is the truest, the querist may judge as he thinks fit."

And so say I.

A readable volume might be compiled from these "Notes and Queries," which amused our grandfathers; and the works I have indicated will afford much curious matter in etymology, folk-lore, topography, &c., to the modern antiquary.

XVIII.

I. ii. 151. **Alarm.**—A man is indicted for striking at the Queen, with intent (among other things) to *alarm* her Majesty. It turns out that the very judge has forgotten the legal (which is also the military) meaning of the word. An alarm is originally the signal to arm: Query, Is it not formed from the cry *à l'arme,* which in modern times is *aux armes*? The judge said that from the courage of her family, most likely the Queen was not alarmed, meaning, not frightened. But the illegal intent to alarm merely means the intent to make another think that it is necessary to take measures of defence or protection. When an *alarm* is sounded, the soldier who is *not* alarmed is the one who would be held to be frightened. M.

To this CH. (p. 183) replied that *à l'arme,* instead of *aux armes,* adds to the suspiciousness of this derivation, and that as corruption generally shortens words he cannot help having the notion that *alarum* was the original form.

"C" (p. 220) states that all French philologists agree as to *à l'arme.* He further notes that in the sense of "an awakening cry" Shakspeare uses *alarum, e.g.* "the alarum bell" (*Macbeth*); "murder alarum'd by his sentinel the wolf" (*Macbeth*); "an alarum to love" (*Othello*); "my best alarum'd spirits" (*King Lear*); in all of which cases *alarum* implies "incitement."

"L" (p. 252) thinks that *alarum* for *alarm* is due to mispronunciation arising from the difficulty of pronouncing *l* and *r* before *m,* and reminds us of the Irish *callum, firrum, farrum,* for *calm, firm, farm,* and of the old English *chrisom* for *chrism.*

"M" replies (p. 252): *Alarm.*—It is in favour of the derivation *à l'arme* that the Italian is *allarme*; some dictionaries even have *dare all'arme,* with the apostrophe, for to give alarm. It is against it that the German *Lärm* is used precisely as the English *alarm.* Your correspondent CH. thinks the French derivation suspiciously ingenious: here I must differ; I think it suspiciously obvious. I will give him a suggestion which I think really suspiciously ingenious; in fact, had not the opportunity occurred for illustrating ingenuity, I should not have ventured it. May it not be that *alarme* and *allarme* is formed in the obvious way, as *to arms*; while *alarum* and *Lärm* are wholly unconnected with them? May it not sometimes happen that, by coincidence, the same sounds and meanings go together in different languages without community of origin? Is it not possible that *larum* and *Lärm* are imitations of the stroke and subsequent resonance of a large bell? Denoting the continued sound of *m* by *m-m-m,* I think that *lrm-m-m-lrm-m-m-lrm-m-m,* etc., is as good an imitation of a large bell at some distance as letters can make. And in the old English use of the word, the alarum refers more often to a bell than to anything else.

The introduction of the military word into English can be traced, as to time, with a certain probability. In 1579 Thomas Digges published his *Arithmeticall Miltare Treatise named Stratioticos,* which he informs us is mainly the writing of his father, Leonard Digges. At p. 170, the father seems to finish with "and so I mean to finishe this treatise:" while the son, as we must suppose, adds p. 171 and what follows. In the father's part the word *alarm* is not mentioned, that I can find. If it occurred anywhere, it would be in describing the duties of the *scout-master*; but here we have nothing but *warning* and *surprise,* never

alarm. But in the son's appendix, the word *alarme* does occur twice in one page (173). It also occurs in the body of the *second* edition of the book, when of course it is the son who inserts it. We may say then, that, in all probability, the military technical term was introduced in the third quarter of the sixteenth century. This, I suspect, is too late to allow us to suppose that the vernacular force which Shakspeare takes it to have, could have been gained for it by the time he wrote.

The second edition was published in 1590; about this time the spelling of the English language made a very rapid approach to its present form. This is seen to a remarkable extent in the two editions of the *Stratioticos*; in the first, the commanding officer of a regiment is always *corronel,* in the second *collonel.* But the most striking instance I now remember, is the following. In the first edition of Robert Recorde's *Castle of Knowledge* (1556) occurs the following tetrastich:

"If reasons reache transcende the skye,
Why shoulde it then to earthe be bounde?
The witte is wronged and leadde awrye,
If mynde be maried to the grounde."

In the second edition (1596) the above is spelt as we should now do it, except in having *skie* and *awrie.*

[The *N.E.D.* gives: O. Fr. *alarme*, a. It. *allarme* = all'arme! 'To (the) arms!' orig. the call summoning to arms, and thus, in languages that adopted it, a mere interjection; but soon used in all as the *name* of the call or summons. Erroneously taken in the 17th c. for an English combination *all arm*! and so written; cf. similar treatment of *alamode* and *alamort.* From the earliest period there was a variant *alarum* due to rolling the *r* in prolonging the final syllable of the call, now restricted to an alarm signal, as the peal or chime of a warning bell or clock, or the mechanism producing it.]

XIX.

I. ii. 199. **Flourish.**—We are told that a writer *flourished* at such and such a time. Is any definite notion attached to this word? When it is said of a century there is no difficulty; it means that the writer was born and died in that century. But when we are told that a writer flourished about the year 1328 (such limitation of florescence is not uncommon), what is then meant? What are we to understand he did in or about 1328? M.

[*D.N.B.* gives

1378. Trevisa *Hyden* (Rolls), IV. 173.
In his tyme Plautus Latinus . . . flourisched at Rome.

1550. Veron., *Godly Sayings,* A. ij.
Origene . . . did florysshe in the yere of our lorde, cc.lxi.

1661. Bramhall, *Just. Vind.* i 3.
His . . . ancestours flourished while Popery was in its Zenith.

1855. Tennyson's *Brook,* l. 11.]

(*To be continued.*)

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58. We have had four sermons from Lemma Vince, the most strange things ever preached in pulpit. The first Sunday he took us thro' the three laws of motion and Wood's chapter on projectiles. [James Wood's *Principles of Mathematics*, 1796.]

The second he got us into his complete system of astronomy [1797]. The third he took us through the 11th section of Newton, and to conclude gave us a dissertation on optical glasses. His pulpit lectures are now ended.

[Samuel Vince was a Suffolk bricklayer, who was Senior Wrangler in 1775; Professor of Astronomy, 1796.]

REVIEWS.

Elements of Graphic Dynamics. By E. S. Andrews. Pp. 189. 10s. 6d. net. 1920. (Chapman and Hall.)

The author must be commended for issuing this volume: as he says in his preface, "The application of graphic methods . . . under the name of 'Graphic Statics' has been very fully developed, but little attempt appears to have been made to develop dynamic problems . . ." along the same lines. The aim of his book, then, is at once a plea for a more general use, in a more logical sequence, of the particular methods known severally to teachers of Practical Mathematics and Applied Mechanics, and to supply a text-book that shall give to each of these classes of teachers the methods known in most cases only to the other class.

I consider this little book so important that I have dwelt upon the few points which seem to me to mar it; and strangely enough, these points occur in what to my mind is the best portion of the book, the first three chapters. These chapters deal with what the mathematician usually calls 'derived' and 'integral' curves for Cartesian coordinates; and it is good to see that the author begins by proving the inverse nature of what he calls the 'slope' and 'sum' curves. I consider that the term 'slope' (which is really the angle of inclination of a curve at any point to the horizontal axis) should have been replaced by the correct term, 'gradient'; and I think that the author's indiscriminate use of the letter x to stand for all sorts of lines, such as an ordinate (p. 2) or an element of the x-axis (p. 6), is unhappy. Another good point is that the author shows by means of concrete examples the usefulness of his subject—the fact that the sum curve of force plotted against distance is a curve of work leading immediately to 'indicator' diagrams; while to Chapter II. is deferred the discussion of Space, Velocity and Acceleration curves on a Time base, which usually form the starting-point of books on practical calculus, and even of some theoretical texts. The section is admirably treated on the whole; it winds up with an exceedingly clear explanation of "Pröll's construction" for obtaining an acceleration graph from a velocity curve on a distance base, and the converse construction in two different ways. There are, however, two points of criticism to be made. On p. 19, we find it stated that "no construction is known which is really more accurate than drawing a line by eye to touch the curve." Of course much depends on what is meant by 'accurate'; but I venture to say that not three students out of a class of twenty can draw a tangent by eye to a four-inch circle so accurately that the perpendicular will pass within a twentieth of an inch from the centre; whereas if two equal arcs are marked off from the point, and a line is drawn parallel to the chord the exact tangent, is obtained. For other curves, this method may be used on the idea of the circle of curvature; but there are grounds for advocating another, the usual method of the practical mathematics teacher, one which also has been suggested as the fundamental principle of theoretical calculus for beginners. That is to say, in the latter case, the tangent may be considered to be given by $\{f(x+h)-f(x-h)\}/2h$ with greater accuracy than by $\{f(x+h)-f(x)\}/h$, when these expressions are given their limiting values. This is all the more conformable to the ideas of the student of practical calculus; for he is taught to depend on the great accuracy of "Simpson's Rule," and the basis of this is that any three points close together on a curve may be considered to determine a parabola with a 'vertical' axis whose arc coincides with that part of the curve between the two extreme points chosen; and this idea gives a practically accurate construction for the tangent, *i.e.* with an accuracy that is of the same order of approximation as that given for areas by Simpson's Rule. For if the three points chosen are the intersections of equidistant ordinates, the tangent at the middle point on the hypothetical parabola is accurately parallel to the chord joining the other two points. The statement is all the more strange because this idea is the fundamental principle of the 'sum curve' as given in Fig. 2 on p. 3. Indeed, p. 19 is altogether unfortunate; for in the graph the tangent representing maximum velocity is

obviously wrongly placed, having been drawn by eye, as the tangent at the point of contrary-flexure, some distance below C, has obviously a greater gradient. Again, on p. 29, the author states "there is no simple construction which gives the slope curve accurately." Such a construction has been given by Prof. Gibson and others, and it should be obvious that there is one that finds the slope curve from the sum curve with as great accuracy as the sum curve is found from the slope curve, namely, by simply reversing the process of construction in the latter case.

Chapter IV. deals with polar diagrams, and opens with a proof that the polar subnormal represents the velocity in a polar space-time graph. Applications are given to Harmonic Motion, Valve-diagrams and Cams; but a general inverse construction is not given, although such a construction was given by Isaac Barrow in 1670. Chapter V. deals with the Hodograph and its application to centrifugal force, and efficiency of turbines. Chapter VI. discusses Instantaneous Centres applied to linked frameworks, quick-return motions, and (of great importance and very clear exposition) the better known constructions for accelerations and velocities of crank mechanisms, the geometrical proofs being, however, somewhat unnecessarily long. Chapter VII. gives the application of Virtual Velocities combined with Instantaneous Centres, and Chapters VIII. and IX. deal respectively with fly-wheel design, and the balancing of rotating parts. The book concludes with groups of useful exercises on each section.

Mr. Andrews seems to me to have produced an exceedingly useful and stimulating volume, but its value is slightly marred by the points noted in the first three chapters, which should be rewritten for that second edition for which, I venture to think, there will be a demand as soon as the value of this admirable contribution to the literature of the subject is generally recognised.

J. M. Child.

The Elementary Differential Geometry of Plane Curves. By R. H. Fowler. Pp. viii + 105. 6s. 1920. (Cambridge Tracts in Mathematics and Mathematical Physics, No. 20. C.U.P.)

The author is to be congratulated on having increased the extremely small number of English mathematical works indispensable to the live student and teacher. Judged by the chapter-headings, the ground covered is familiar enough—tangents and normals, curvature, contact, envelopes, singular points, asymptotes; but there is a century of mathematical progress between the arguments with which previous English writers have been content and Mr. Fowler's presentation modelled on the best French work.

The treatment is extrinsic, rectangular Cartesian axes being used, and the curve taken usually in the form $y=f(x)$. Attention is practically confined to real space; not only are the curves considered supposed to be real, but there is no mention of imaginary points of intersection, of circular points or isotropic tangents, or even of points at infinity. The distinctive feature is the application of the notation now accepted for dealing with magnitudes tending simultaneously to zero or to infinity; every formula embodies an assertion that is perfectly precise. This does not imply that increments and differentials are not used; on the contrary, both are employed freely, and the distinction between them is observed. But infinitesimals of course are banished, and with them the "consecutive points" and "consecutive tangents" by means of which wrong results can always be made at least as plausible as right ones. Great pains have been taken with the definitions, and it is a valuable part of the author's plan, especially with regard to tangents, to circles of curvature, and to asymptotes, to discuss to what extent different definitions in use are in fact equivalent.

The section most enlightening to the student accustomed to the rough and ready manners of an earlier day is that on envelopes, written with a scrupulous adherence to logic that is beyond all praise. Having been warned in the preface that in the current definition no curve is the envelope of its own circles of curvature, the reader will not expect to come across the "intersection of consecutive members of the family". Instead, he finds that the point M is to be on the envelope if, to put it roughly, (1) curves of the family

are unnaturally close together near M, (2) the curve through M is not composed wholly of points with the first property. Unfortunately, this definition involves difficulties to which no reference is made: a number of points that one would certainly like to find on the envelope have to be rejected (for example, the envelope of the tangents to $y=f(x)$ is not the whole curve, but the curve minus the points * at which $f''(x)=0$); the envelope depends not on the family alone, but on the parameter by which the individual curves are identified, and there is no point on the envelope which a change of parameter would not enable us to banish! *These* troubles would not arise if the distinctive feature was taken to be a concentration near M, unnatural compared not merely with that near an ordinary point of the plane, but with that near other points *of the curve through* M; but it will require care to discover what troubles this definition would introduce that the one adopted in the tract avoids.

The discussions of the form of an algebraic curve near a singular point and of the asymptotes, straight and curved, of such a curve are admirable and complete in themselves, but use might have been made of Newton's parallelogram. Rule III. for Asymptotes (p. 101) is unduly restricted in scope: a trivial modification gives a rule applicable when some of the asymptotic directions are not known, and when the number of real asymptotes falls short of the degree of the curve.

Doubtless it was great fun to interpolate favourite fragments of solid geometry †, but it is perhaps a pity that Mr. Fowler yielded to the temptation. The space could well have been occupied with two-dimensional matter, for example, with some account of the relations between a function of position and the families of curves associated with it: the ordinary expression given on p. 26 for the curvature of $F(x, y)=0$ is singularly unintelligible, regarded only as an analytical transformation of the expression for the curvature of $y=f(x)$. Moreover, there is ample room for a tract dealing with twisted curves and ruled surfaces systematically in the manner of this one, and it is too bad of Mr. Fowler to have skimmed so much of the cream!

E. H. N.

Mathematics for Engineers. Part I. By W. N. Rose. Pp. xiv + 510. 8s. 6d. net. 1918. (Chapman & Hall.)

Theory of Structures. By A. Morley. Pp. xi + 584. 14s. net. 1918. (Longmans.)

The D.U. or "directly useful" Technical Series is a new venture on the part of the publishers, under the editorship of Mr. W. J. Lineham. "Directly useful" seems to imply a contrivance to pay a double debt. Objection is raised to the technical book which has been written "more for the training of college students than for the supply of information to men in practice." Moreover the examples set in such books have been in the main of an unreal type. On the other hand, the so-called practical book has tended to be little more than a catalogue of formulae with hints as to their use. The D.U. steps into the breach, gives information, problems, and exercises dealing with the real things of the engineering world, but all this is "at the same time wedded to that proper amount of scientific explanation which alone will satisfy the inquiring mind." In the first chapter, on aids to calculation, we come to the editor's own device for approximations, with the accompanying pabulum for the "enquiring mind." We give it verbatim with a worked-out example. Those who do not approve, may improve.

"Reduce each number to a simple integer, *i.e.* one of the whole numbers 1, 2, 3, etc., if possible choosing the numbers so that cancelling may be performed; this reduction involving the omission of multiples or sub-multiples of 10. To allow for this, for every

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* I should like to say "the points of inflexion," but Mr. Fowler's definition does not allow me; may I suggest that to be consistent Mr. Fowler ought to *define* a tangent as a line that meets a curve without crossing it!

† Readers should be warned that writers differ in the convention governing the sign of torsion, and that opinion is settling down in favour of the opposite rule to that used in the tract.

'multiplying 10' omitted place one stroke in the corresponding line of a fraction, spoken of as a *point* fraction, and for every 'dividing 10' place one stroke in the other line of this fraction. Thus two fractions are obtained, the *number* fraction, giving a rough idea of the actual figures in the result, and the *point* fraction from which the position of the decimal point in the result is fixed. Accordingly, by combining these two fractions, the required approximate result is obtained. . . ."

"Ex. 2. Determine the approximate value of $\dfrac{9764 \times 0{\cdot}0213}{28{\cdot}4 \times 0{\cdot}00074}$.

State 9764 as 10,000, *i.e.* write 1 in the numerator of the number fraction and four strokes in the numerator of the point fraction.

For 0·0213 write 2 in the numerator of the number fraction and two strokes in the *denominator* of the point fraction. The strokes are placed in the denominator, because in substituting 0·02 for 2 we are multiplying by 100, and therefore to preserve the balance, we must divide the result by 100.

For 28·4 we should write 3 with 1 stroke in the denominator, and for 0·00074 we should write 7 with 4 strokes in the numerator. Thus:

Number Fraction.		Point Fraction.
$\dfrac{1 \times 2}{3 \times 7}$		$\dfrac{1111\ \ \cancel{1111}}{\cancel{11}\ \ \cancel{1}}$
i.e. 0·1	and	$\underline{11111}$ by cancelling.

Hence the **approximate result is 0·1 × 10⁵, i.e. 10,000**; or, alternatively, the shifting of the decimal point would be effected thus:

$$\cdot 10000 \cdot ."$$

This method is extended next to square and cube roots:

"Reduce the number whose square root is to be found to some number between 1 and 100, multiplied or divided by some even power of ten; then the approximate square root of this number, combined with half the number of strokes in the point fraction, gives the approximate square root of the number.

In the case of cube roots, the number must be reduced to some number between 1 and 100 multiplied or divided by 3, 6, or 9... tens; then the approximate cube root of this number must be combined with one-third of the strokes in the point fraction."

"Ex. 6. Evaluate approximately $\sqrt{\dfrac{21{\cdot}43 \times 0{\cdot}097}{0{\cdot}154 \times 2409}}$.

Disregarding the square root sign for the moment, the approximation gives:

$\dfrac{2 \times 1}{1{\cdot}5 \times 2}$	$\dfrac{1\ \ \cancel{1}}{1\ \ 1\cancel{11}}$
i.e. 0·67	

For the application of the method of this paragraph this result would be written:

67	$\overline{1111}$

of which the square root is

8·2	–	$\overline{11}$

or the approximate square root is **0·082**."

We are told that this method "has been found to be very effective and easily grasped; and it has been felt that the device here described for investigation for units could be more universally employed, because of its simplicity and directness."

The section on graphs and their applications are simple and clear, and their practical character may be gauged from such pages as those devoted to questions such as the allocating of allowance for depreciation of plant.

"A departure is made from the old convention of the measurement of angles from a horizontal line, calling them positive if measured in a counter-clockwise direction Plane Trigonometry has its widest application in land surveying, in which angles are measured by a right-handed rotation from the north direction. Hence the north and south line is here taken as the standard line of reference, and all angles are referred to it. Also by doing this the mathematical work is simplified, since the trigonometrica ratio of an angle does not alter; the angle of any magnitude being converted to the 'equivalent acute angle,' viz. the acute angle made with the north and south line. The

calculation of co-ordinates in land surveying is introduced as a good instance of the solution of right-angled triangles. . . ." *

From graphic integration and the use of the planimeter we pass in chap. viii. to useful worked-out examples on the calculation of volumes of earthwork, cost of removal of earth from excavations, and the like. The "Plotting of Difficult Curve Equations" extends to plotting of diagrams for primary and secondary adiabatics, for jacketed and non-jacketed engines, and the PV and $\tau\phi$ diagrams for various engine types. Another useful chapter is that on the principle and use of alignment charts. The author seems to have selected his examples with care, and there are not too many of them. Part II. is to deal with the Calculus and its applications to engineering. Teachers in secondary schools will do well to consult this volume. Much may be learned from the experience of those who have had to instruct individuals at various ages and in varying stages of intellectual development. And we all wish to remove the reproach that our problems are too academic. The book is clearly printed in large type, and the diagrams are plainly and well drawn.

We have left ourselves no space to dwell upon Mr. Morley's new edition of the *Theory of Structures*. It does not seem to differ from the edition of 1912, and the author's merits as a teacher of outstanding merit have long been recognised.

Problems in Dynamics. (With Full Solutions.) By Atma Ram. Pp. 245. 3 rupees. n.d. (To be had from the Author, Asst. Prof. of Maths., Government College, Lahore, India.)

Mr. Atma Ram's collection of fully worked-out problems in Elementary Dynamics covers the ground mapped out for the Pass and Honours B.A. of the Indian Universities. In other words, it comprises the sections in the usual English text-books on the subject, and has, in addition, two chapters on: Central Forces, Law of the Direct Distance; Central Forces, Planetary Motion. The examples selected are of varying difficulty, and a large number are familiar to English teachers, being originally in Tripos, Scholarship, and the easier examination papers. The compiler has obviously taken great pains in their selection, so that the student who has thoroughly grasped the method of solution in each case may quite fairly be said to have a "mastery" of the dynamical principles underlying any problem of the same type that may again present itself. But so clear is Mr. Ram's exposition, and so neat and elegant are a large number of his solutions, that the volume may also be useful to other than private students if utilised under the wise guidance of a teacher who can trust his pupils to follow his counsels. The various misprints and slips which seem as yet inseparable from such productions of Indian firms are not of a very serious nature, and should not impede the progress of a student of average capacity.

The Elements of Physics. By R. A. Houstoun. Pp. viii + 221. 6s. net. 1919. (Messrs. Longmans, Green.)

The ground covered in this excellent introduction to Physics is that required for the first Professional Examination in Medicine. It may be placed in the hands of such beginners as have made their bow to the elements of trigonometry. The divisions are: Dynamics, pp. 1-40; Hydrostatics, 41-70; Heat, 71-105; Sound, 106-127; Light, 128-161; Electricity and Magnetism, 162-209.

Mathematical Papers for Admission into the Royal Military Academy and the Royal Military College, and Papers in Elementary Engineering for Naval Cadetships, for the years 1909-1918. Edited by R. M. Milne. 7s. 1910-1919. 10s. 6d. (Macmillan.)

Invaluable for Army and Navy Classes.

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* Since the above was written, we have noticed that this standard line, which is used in the Gyro-compass system of bearings, is adopted by Mr. H. E. Piggott in his *Elementary Plane Trigonometry* (Messrs. Constable).

Lectures on Ten British Physicists of the Nineteenth Century. By A. Macfarlane, pp. 114. 5s. 6d. net. 1919. (John Wiley, Chapman & Hall.)

The lectures published under this title are conceived on the same scale as the *Ten British Mathematicians*, already noticed in the columns of the *Gazette* (vol. ix, no. 131, pp. 146-152). Some readers may think that the line between "mathematician" and "physicist" has in some cases been oddly drawn, but it is idle to discuss the point in a posthumous work. Much more attention has been paid to the proofs of this volume than was received by its predecessor. "Lionville" is familiar enough, but "Fontenall" is a novel variant of Fontenelle: and in a Briton's lucubrations we do not expect to see "Brittanica." However, these appear to be the only slips, if "Menabréa" is correct, but we are afraid it is not.

The first of the lectures is on Clerk Maxwell, "the clearness of whose mental vision," in the words of Tait, "was on a par with that of Faraday," and thus either by accident or of set purpose, pride of place is given to "one of the greatest creative geniuses of all time." The lecture was delivered some eight years after the publication of the *Life* by Campbell and Garnett, so that, with his own recollections, the lecturer had ample material at his disposal.

The subject of the next is W. J. Macquorn Rankine, who shared with Thomson and Clausius the honour of founding theoretical thermodynamics. He was a born mathematician, an accomplished engineer, a distinguished naval architect, and a poet like Maxwell—was he not, indeed, "King" of that merry band of British "Asses" who took the name of "Red Lion"? Science was bereft of both these Scots at an early age, of the one at 48 and of the other at 53. To what Macfarlane tells us of Rankine we may add the concluding words of the obituary notice written by John Mayer:

"Whatever he wrote he executed with almost matchless perfection, whether we regard the elegance of his diction, the scientific order of his exposition, or the lucid methods of exposition which he adopted. His mind was of the very first order, and his death creates such a profound void in pure physics and scientific engineering that we could easily have afforded to give half-a-dozen of our most eminent practical engineers, civil or mechanical, that he might have been retained among us to pursue his original investigations, and mould the minds of the engineers of the future."

In Tait and Thomson, we have two more Scots, the T.[1] and T. of Maxwell. Macfarlane had not the advantage of reading the classical biographies of these men—the one by Dr. Knott and the other by the late Silvanus Thompson. The story of the father of the champion golfer as given on p. 52 is told otherwise by Dr. Knott. "Freddie" Tait's "glorious drive of 250 yards 'carry' on a calm day"—given in *The Times* for May 4, 1911, as 390 yards, was in no way a disproof of his father's supposed dictum as to the maximum distance to which a ball can be driven. But for this *v.* Dr. Knott's fascinating *Life*, pp. 25→.

It is easy to see whence Macfarlane derived his interest in Quaternions, and he writes at his best when he writes of Tait the man; of Thomson's "grand man whom it was a privilege to know"; of the quiet humourist who said that he "could coach a scuttle to be a Senior Wrangler"; and who said: "it is when you are filling your pipe that you think your brilliant thoughts."

Truly he was, to quote Flint, "among the rare few in a generation of whom the memories live on through the centuries," and there must be men living yet who will recognise Tait from Chrystal's description: "ten minutes in whose sanctum would make a friend of his bitterest foe." When a pupil in Tait's laboratory, Macfarlane had an opportunity of seeing something of Kelvin, then Sir William Thomson.

"He must originally have been about six feet high. But for many years his height has been diminished by a stiff leg which was brought about in the following way. He broke his leg when skating on the ice, and would not remain at rest until it had recovered properly. . . . Compared with Tait, he was not so elegant a speaker, but his papers have more of the stamp of a genius. He has strong opinions of most subjects and, like most Irishmen, he is not afraid of a controversy. . . . On social matters he has strong conservative opinions; at a club meeting, after the regular meeting of the Royal Society of Edinburgh, he was asked: 'Sir William! What do you think? Should a man be allowed to marry his widow's sister?' 'No, sir, the Bible forbids it, and I hope the law of the land will continue to forbid it.'"

Sir William, or Macfarlane, might have indicated the passage in Holy Writ in which the impropriety of such a union is intimated.

The next of the "physicists" carries us back to the days of Peacock and Herschel, to a character for whom there is less biographical material at our disposal, to Charles Babbage, embalmed in the (Ingoldsby) "Lay of St. Cuthbert" as

> "Master Cabbage, the steward, who'd make a machine
> To calculate with, and count noses,—I ween
> The cleverest thing of the kind ever seen. . . ."

Babbage wrote little but fragments and his autobiographical *Passages in the Life of a Philosopher* is no exception to the rule. There we find all that is known of his early days. He was at school with Marryatt, the future novelist, and his first taste for mathematics came from Ward's *Young Mathematician's Guide*. The earliest recorded sayings of children are not always a key to their future careers. I know nothing earlier in Kelvin's life than his saying, when caught in front of a glass: "Pitty b'ue eyes Willie Thomson got." But there is a strong resemblance between Maxwell and his cry of: "what's the go o' that"? or, when dissatisfied with the reply, his further demand: "what's the particular go o' that?", and Babbage, whose every new toy was welcomed with: "Mamma, what's inside of it"? and his increasing impatience until he had broken it open and could gratify his curiosity for himself. As he grew up he soon longed to devour any work on mathematics that came his way. He fell in with works on fluxions by Maclaurin, by Simpson, by Humphrey Ditton—a name that takes us back to Whiston and Swift—with the *Analytical Institutions* of Madame Agnesi—probably the Colson-Hellins translation of 1801,—with Lagrange's *Théorie des Fonctions*, and above all with Woodhouse's *Principles of Analytical Calculation*, 1803. This was the book that, as De Morgan says, heralded the revolution, and in due course, the souls of the Cambridge moderators, as Herschel picturesquely put it, "though fenced with seven-fold Jacquier, and tough bull-hide of Vince and Wood," were touched to some extent. The movement was watched by continental mathematicians with a kindly interest, and when Herschel and Babbage paid a visit to Laplace they were greatly puzzled by his frequent reference in the warmest terms to "un ouvrage de vous deux." Not having collaborated they looked puzzled, so Laplace corrected himself to "de vous deusse," which was indeed a gallant attempt at the name of the author of the *Treatise on Isoperimetrical Problems*.

It was natural that Babbage should find difficulties in his reading, and, innocently enough, he looked forward to going up to the University to have them removed.* We read with an understanding sympathy that, on his way to Cambridge, he had as much as seven guineas to pay for a copy of Lacroix's *Differential Calculus*, as "during the war it was difficult to procure foreign books." The first part of this he translated and published in 1816, Peacock and Herschel translating the second part (*Integral Calculus*). In the first week of his University life he applied to his Tutor for an explanation of some of his difficulties, and was politely told that such questions would not be asked in the Senate House. He tried a lecturer, and then another authority, but all enquiries were met with the same narcotic indifference. The result he expresses characteristically: "I thus acquired a distaste for the routine of the studies of the place." The volume in which Babbage and his friends sounded the signal of revolt deserved the title he suggested—"The Principles of Pure D-ism in opposition to the Dot-age of the University." Among those who with Babbage founded the Analytical Society were Sir Edward Ffrench Bromhead (to whose interest is due the appearance of the name of George Green two places below that of Sylvester in the Tripos list of 1837 and on the roll of Fellows of Caius), and d'Arblay, the only son of Johnson's beloved Fanny Burney. D'Arblay might well have been a New-

* Paternal anxiety had led the father to consult a Fellow and Tutor of a College in the hope of picking up some hints that would be useful to the young undergraduate in his University career. But the only information to be extracted from this "lord of human kind," with "pride in his port" "and defiance" (in the French sense) "in his eye," was: "Advise your son not to purchase his wine in Cambridge."

tonian by sentiment, for his grandfather lived in No. 36 St. Martin's St., the house in which Newton lived and died.

Babbage tells us little more of Cambridge and his undergraduate life. He migrated in his last year from Trinity to Peterhouse, and, being certain that his friend Herschel would get the first place in the Tripos, he contented himself with a pass degree, being at the time full of his new Difference Engine, which, with its successor, was to be to him such a source of pleasure and disappointment. His degree did not prevent his election to Newton's chair in 1828—"the only honour I have received in my own country" (and he significantly adds "this professorship is not in the gift of the Government"). In those days the emoluments of the Chair were between £80 to £90 a year, but neither lectures nor residence were essential parts of the duty of a Professor. Honours bestowed by Government on men of science were few and far between—a Hanoverian knighthood among the doctors and other scientific men of the day was considered quite enough in the way of distinction for such insignificant persons. And such honours as that have a way of becoming estimated at their real value.* One great Queen, whom three realms obeyed, did herself the honour of giving the accolade to Sir Isaac Newton at Cambridge, but the next Queen who visited the University found that the offer of a knighthood was declined by Adams, as it had been by Airy twelve years earlier. Faraday was a Commander of the Legion of Honour and a Knight Commander of the Order of St. Maurice and Lazarus, but apparently was offered no English honour of the kind. Ivory took the pension attached to the Guelphic Order, but either never used or soon dropped the title. Playfair did his best to obtain for both Babbage and Faraday the honour of being Privy Councillors, but his efforts were in vain. So perhaps we can understand the melancholy reflection of Babbage—"the only honour I have ever received in my own country." Melbourne—who loved the Garter because "there is no damned merit about it"—expressed himself very frankly (and afterwards apologised for it) about the system of giving pensions to scientific persons, calling it a piece of humbug. It is interesting to remember that Faraday's own suggestion was that an order to which scientific eminence alone should be a passport would probably have a happy effect upon the progress of science in this country. "The sole order of nobility which becomes a philosopher is the rank which he holds in the estimation of his fellow-workers, who are the only competent judges in such matters. Newton and Cuvier lowered themselves, when one accepted an idle knighthood, and the other became a baron of the empire. The great men who went to their graves as Michael Faraday and George Grote seem to me to have understood the dignity of knowledge better when they declined all such meretricious trappings." —*Huxley*.

Babbage intended to lecture as part of the duties of his Chair, and the material he collected for the purpose was published in a volume dedicated to the University, and entitled *Economy of Manufactures and Machinery*. Of all his printed productions he was proudest of this—his "hymn in honour of machinery." It seems to have produced an impression in the most unexpected quarters, for Rogers the poet told him that a young dandy had described it as "just the sort of book that anyone might have written," while a workman had told the author that he loved the book, because it made him think. In its pages the possible decay of the coal-fields is discussed, and it is suggested that before their complete exhaustion is approached a suitable substitute will be devised.

"The sea itself offers a perennial source of power hitherto almost unapplied. The tides twice in each day raise a vast mass of water which might be made available for driving machinery. . . . In Iceland. . . . the sources of heat are plentiful. . . . and their proximity to large masses of ice seems almost

* In 1831 the Guelphic Order of Knighthood was bestowed on Sir Charles Bell (anatomist), John Leslie (Math. and Nat. Phil.), J. F. Herschel and James Ivory. Herschel's father, William, was made K.H. in 1816. Before the death of William IV., only two physicians—Sir Henry Halford and Sir Matthew Tierney—and one surgeon, Sir Astley Cooper—were made G.C.H. To Dr. Chambers, who attended William IV. on his deathbed, and to Dr. David Davies, who had been medical attendant to William when Duke of Clarence and to the end, the King of Hanover gave the cross. Chambers declined knighthood.

to point out the future destiny of that island. The ice of its glaciers may enable the inhabitants to liquefy the gases with the least expenditure of mechanical force, and the heat of its volcanoes may supply the power necessary for their condensation. Thus in a future age *power* may become the staple commodity of the Icelanders, and of the inhabitants of other volcanic districts; and possibly the very process by which they will procure this article of exchange for the luxuries of happier climates, may, in some measure, tame the tremendous element which occasionally devastates their provinces."

Our lecturer clearly sketches the story of the engines, and carefully distinguishes the Difference Engine from that Analytical Engine to which the inventor devoted 37 years of his life and a goodly part of his fortune. It was fortunate that Babbage was little troubled by financial anxieties. He lived, indeed, in some state, and in his house might be seen the cream of the intellectual society of the day. Thus we read in Ticknor's *Journal* that he took "Ion" Talfourd to Babbage's, "where there was a grand assembly, lords and bishops in plenty." . . . And again, "About eleven o'clock we got away from Lord Fitzwilliam's and went to Mr. Babbage's, who at this season gives three or four successive routes on successive weeks. It was very crowded to-night, and very brilliant; for among the people there were Hallam; Milman and his pretty wife; the Bishop of Norwich [this was the "bird-catcher," the father of Dean Stanley]; the Bishop of Hereford [Musgrave, Cambridge Professor of Arabic and later Archbishop of York]; Rogers [the poet]; Sir J. Herschel and his beautiful wife; Sedgwick; Mrs. Somerville and her daughters; Senior; the Taylors [probably Sir Henry, author of *Philip van Artevelde*, and his wife]; Sir F. Chantrey; Jane Porter [authoress of *Thaddeus of Warsaw*]; Lady Morgan [novelist, etc., very unkindly handled by the *Quarterly*]; and I know not how many others."

Edward Bulwer Lytton in 1838 drives from Holland House "to Babbage's, to see all the world and his wife"; Crabb Robinson, a connection of De Morgan, and the first of war correspondents, dining with the future Baroness and philanthropist, Miss Coutts, found among the guests Gleig, who wrote under a name that has appeared again of late years—"Subaltern"—and who was Chaplain General to the Forces; Sir Charles Napier of "Peccavi" fame; Peter Barlow, author of a "Theory of Numbers," who had now retired from the "shop" some five years; and the "militant Babbage."

"Militant Babbage!" He also struck Lord Houghton as "always at white heat, ready to scorch up some rival man of science." At the Royal Society dinner, March 21, 1822, Hobhouse "sat next to the great arithmetician. He remarked to me that he had observed how much mechanical skill had of late taken the direction of improving printing. He told me that a tolerably-sized 8vo vol. cost, as far as types and ink were concerned, only 8d.!" Twenty years later they dined together, this time at the Duke of Somerset's. There is nothing of Babbage's conversation to record on this occasion, but the story of how Sir John Pringle came to resign the Presidency of the Royal Society was told by the Duke, and is too good not to repeat for the sake of those who have not heard of the "balls and points" controversy. The King, George III., was talking to Pringle of Franklin's discovery, that the attraction of a pointed body is greater than that of a level or circular surface; and his Majesty expressed a wish to have it contradicted. "Sir, I cannot alter the laws of nature." "Then," said the King, "you are not fit to be President of the Royal Society." "This was the way the Duke told me the story; but I thought it almost too good to be true," says Hobhouse (*Broughton*, vi. p. 55). The year before they had dined at Lord Lansdowne's, when "our talk was lively and discoursive, Babbage defended monopolies, and said that the interest of the monopoliser would operate as favourably for the public as competition." Austin and Macaulay took the other side in the discussion. Macready, who seems to have lived in a frenzy of bitterness at the social successes of others, writes in June 1834: "I am sorry to hear from Lardner of the childish weakness of Babbage, and his base anxiety for the notice of titled persons. In a mere song-writer like Moore, notorious for this, it is not surprising, but in a man of science we blush for the character of the philosopher."

It is well to record that the "militant" philosopher was the first to suggest that a pension should be awarded by the State to Dalton. This was in 1829, and Lord Brougham professed himself very anxious to secure such a public recognition in so deserving a case. At last he reported that it "was attended with great difficulty," though just at this time he had found no difficulty in making his own brother a Master in Chancery, and then introducing a Bill to abolish the office and to give retiring Masters £2200 a year. However, in 1833 £150 a year was granted, and the fact announced by Sedgwick in his British Association address of that year.

Sedgwick's comment on this was: "Talleyrand called England a paradise of priests, but he might have said a paradise of lawyers, when he knew that a fourth-rate almost briefless barrister was measured as 2200 to 150 to the greatest man of science since Newton."

Three years later the pension was increased to £300. Babbage was present when Dalton went to Court in his scarlet Oxford D.C.L. gown (or was it his Edinburgh LL.D.? in any case "Daltonism" records that it was green to Dalton's eyes), and overheard the conversation between William IV. and the quiet Quaker. The Peterloo massacre had taken place some 15 years before, but all that the Sailor King said to his subject was: "How are you getting on at Manchester? all quiet, I suppose!" The answer was direct enough: "Well, I don't know: just middling, I think."

Babbage was always willing to explain to enquirers the working of his beloved engines. A layman's account is not without interest. Ticknor, July 12, 1835, thus describes such a visit:

"From Church we went, by his especial invitation, to see Babbage's calculating machine; and I must say, that during an explanation which lasted two or three hours, given by himself with great spirit, the wonder at its incomprehensible powers grew upon us every moment. The first thing that struck me was its small size, being only about two feet wide, two feet deep, and two and a half high. The second very striking circumstance was the fact that the inventor himself does not profess to know all the powers of the machine; that he has sometimes been quite surprised at some of its capacities; that without previous calculation he cannot always tell whether it will, or will not work out a given table. The third is that he can set it to do a regular operation, as, for instance, counting 1, 2, 3, 4; and then determine that at any given number, say the 10,000th, it shall *change* and take a different ratio, like triangular numbers, 1, 3, 6, 9, 12, etc.; and afterwards at any given point, say 10,550, change again to another ratio. The whole, of course, seems incomprehensible, without the exercise of volition and thought. . . . But he is a very interesting man, ardent, eager, of almost indefinite intellectual activity, bold and frank in expressing all his opinions and feelings. . . ."

(Ticknor knew little of numbers, and uses "triangular" in a non-mathematical sense). We can sympathise with the lady who, after beholding these wonders, observed: "So there's something after all in a Chinese praying-wheel." Nearly twenty years later, Grant Duff records that he met Babbage.

"He appeared to me one of the most remarkable intelligences I have ever come across, though he wasted his time on all kinds of ingenious frivolities. I think his hobby at this period (1853) was reading cyphers, and he afterward spent endless time on making a cypher dictionary."

Wallis redivivus!

It is interesting also to take a peep behind the scenes. Peel writes to Croker, from Whitehall, in 1823:

"You recollect that a very worthy seafaring man declared that he had been intimate in his youth with Gulliver, and that he resided (I believe) in the neighbourhood of Blackwall. Davies Gilbert [then treasurer and soon to be President of the Royal Society] has produced another man who seems able to vouch at least for Laputa. Gilbert proposes that I should refer the enclosed to the Council of the Royal Society, with the view of their making such a report as shall induce the House of Commons to construct at the public charge a scientific automaton, which, if it can calculate what Mr. Babbage says it can, may be employed to the destruction of Hume [Joseph, the 'pestilential' advocate of economy, a radical M.P.]. I presume you must at the Admiralty have heard of this proposal—

'Aut haec in vestros fabricata est machina muros,
Aut aliquis latet error.'

I should like a little previous consideration before I move in a thin house of country gentlemen, a large vote for the creation of a wooden man to calculate tables from the formula. I fancy Lethbridge's face on being called upon to contribute. . . ."

(To be continued.)

THE PILLORY.

The "annexed figure" of the question may be reproduced from the data: Take a pair of rectangular axes, x-axis horizontal, y-axis vertical (shaded on its left side). Mark the points $A(0,0)$, $B(2,2)$, $C(3,1)$, $D(2,0)$, $E(0,2)$. Join AB, BC, CD, DE, BD. The x-axis should not be shown, and the y-axis should be shaded on its negative side. A downward vertical through C indicates the hanging weight.

In such a figure AB, BC, CD, DE, DB are light rods, smoothly jointed at A, B, C, D, E; A, E being fixed points on a vertical wall. A weight of 50 lb. is suspended from C. The marked angles are each half a right angle. Find the stress in each of the five members, and indicate whether it is a tension or a thrust. [No angles were marked.]—*Inter Sci.*, London, July, 1919. (Per W. J. D.)

Parallel lines AE, CF are drawn through the vertices A, C of a triangle ABC; and from any point in AC perpendiculars are drawn to AB and CB, meeting AE and CF in U and V respectively. Prove that, if UBV is a straight line, the angle ABC is a right angle.—*London University B.Sc. Honours*, 1912.

I am curious to see if this question will invite a comment. G. N. B.

QUERY.

Can any of you readers supply the source of the following line, which used to be quoted by the late Professor Ingram Bywater to his friends?

"Dirus Asymptotes faciesque obscoena Trapezi."

Oxford. FAMA.

ERRATA.

Reader, Carthagena was of the mind that with those three things which the Ancients held impossible, there should be added this fourth, to find a book printed without Erratas. It seems that the hands of Briareus and the eyes of Argus will not prevent them.—Cotton Mather's *Magnalia Christi Americana.*

Note 540, p. 43, vol. x. *For* $x+\{\ \}$ *read* $x\{\ \}$.

For $C_5(1-x)^5$ *read* $^{10}C_5(1-x)^5x$.

[Pointed out by several contributors.]

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